

# What can old firms tell us about the effect of age on firm size

## REVISION OF Firm-level Entry and Exit over the Danish Business Cycle\*

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If old firms are on average much larger than young firms, does it mean that firms get better with age? Using Danish administrative data, we construct age profiles of firm size (measured by the number of workers, turnover and value added) for ages 0-60. We find that the positive effect of aging stops around the age of 10 after which firms start to deteriorate. The positive correlation of size and age is caused by a strong selection effect where firms from the bottom of the distribution are more likely to exit.

JEL codes: D22, E23, E24, L11

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# 1 Introduction

There is a growing literature that documents a fall in firm entry and points out that this has implications for employment: young firms generate the majority of new jobs, so these “missing firms” lead to a reduction in aggregate employment (Davis and Haltiwanger, 2014; Decker et al., 2014; Adelino et al., 2017; Pugsley and Sahin, 2018, e.g.). However, there is also a second effect: lack of entry pushes up the average age of firms in the economy. In this paper we present evidence of the firm life cycle and show that firm size and activity deteriorate after reaching maturity. The fall in firm entry can hence indirectly manifest itself in the deterioration of the pool of incumbent firms.

Using administrative data where we can track firms of all ages, we show that older firms are on average larger than younger firms and very old firms are on average very large. We find that this is the outcome of a very strong selection effect: firms of the highest quality survive and the smallest firms are more likely to exit. At the same time, for a given firm the effect of age is mostly negative and firms generally deteriorate after reaching maturity around a firm age of 10 years.

We are able to analyze firm life cycle profiles up to a firm age of 60, which is much longer than what is typically done in the literature. The reason is that we exploit a Danish administrative micro data set that provides direct information about firm age. In contrast to other data sets used in the literature, the age information is not truncated and hence very old firms can be studied. For example, one particularly useful source of information about firm dynamics has been the Longitudinal Business Database (LBD). However, due to the way the LBD is structured, firm age is not directly observed and hence it is not possible to identify the age of firms that entered before the LBD started. In contrast, Danish registry data contains precise information about the starting date and therefore allows to distinguish the effect of aging even among old firms. Furthermore, our data contains the universe of Danish firms of all sizes and all sectors, while other data sets often only contain publicly traded firms which biases the sample towards the largest firms.

We first show that on average firm size increases with firm age. A 30-year old firm on average is 1.5 times the size of a 10-year old firm (measured as employment, value added and turnover). Moreover, size increases even more strongly at very old ages: a 60-year old firm is on average 3.5-7 times larger than a 30-year old firm. In a second step we use fixed effects regressions to disentangle the causal effect of aging from selection issues. Age could increase firm size by, for instance, building up knowledge about the firm’s competitive

environment, human capital or firm-specific skills. Selection effects, on the other hand, could increase average firm size if smaller firms are systematically more likely to exit. We find that selection effects drive the average firm size: the causal effect of aging is negative for most of a firm's life cycle. Employment, value added and turnover decrease with age once a firm has matured at around 10 years of firm age. At the same time, the average fixed effect in the surviving population, interpreted as inherent firm quality, increases strongly with firm age. Finally, we predict the likelihood of exit as a function of firm size and find support for the hypothesis that small firms are systematically more likely to exit: A firm in the bottom third of the size distribution is on average at least twice as likely to exit compared to a firm of the same age that is in the top third of the size distribution.

There is a long tradition of examining the role of certain firm characteristics such as size and age on its outcomes. In aggregate, changes in the unemployment rate are directly linked to the changes in employment in individual firms and so understanding the characteristics of growing firms is an important question both for academics and for policymakers. This paper contributes to this debate by focusing on the effect of aging. [Evans \(1987\)](#) was among the first to use a comprehensive manufacturing firm data to uncover a negative effect of age on firm employment growth between 1976 and 1980. Relative to this founding study, our data is more granular with respect to age and perhaps due to this fact we are able to uncover a more complex relationship between the age and the odds of exiting. [Haltiwanger et al. \(2013\)](#) find that size does not drive employment growth after controlling for age.

Close to the analysis in this paper, [Navaretti et al. \(2014\)](#) use EFIGE survey combined with Amadeus database for France, Italy and Spain between 2001 to 2008. Compared to our sample, they study only surviving manufacturing firms with more than 10 workers, whereas we analyze firms from all sectors, both surviving firms and those that exit (which allows us to analyze the likelihood of exit as a function of firm size) and without lower bound on employment. They find that firms grow more slowly when they are older. They find that even after including many other variables (such as the age of CEO, number of graduate in workforce, R&D activity, productivity, capital intensity, profitability, finance), the effect of age is still significant. Using a data set of Italian exporting firms, [Grazzi and Moschella \(2018\)](#) find that the positive relationship between export status and growth declines with firm age. In the sample of Irish firms between 1972-2010, [Lawless \(2014\)](#) finds that younger firms grow faster than older firms. Moreover, using the Revenue-enhanced Longitudinal Business Database, [Alon et al. \(2018\)](#) compute age profiles for productivity growth. Compared to results presented in this paper, their highest age group is 11-15,

compared a maximum firm age of 60 in our analysis. The basic pattern of our finding is similar to theirs, but by being able track firms for much longer we are able to document that the deterioration continues even for very old firms.

In this paper we also contribute to the recent debate about the role of shocks versus the role of inherent firm characteristics that are present at the firm entry. We find that the larger average size of older firms is driven by inherent firm quality rather than positive effect of aging. This finding is in line with other recent developments in this area. For example, [Sedláček and Sterk \(2017\)](#) find that firms are heavily affected by the conditions when they start, or [Pugsley et al. \(2017\)](#) show that “...even after twenty years, *ex-ante* factors still explain about forty percent of the cohort’s employment dispersion”. We show the changes in the distribution in the firm inherent quality as firm age.

The paper proceeds in the following steps. Section 2 describes the Danish administrative data set and discusses its advantages over other data sources. Section 3 computes average size, measured as employment, value added, and turnover. The results show that average size is increasing with age. Using fixed effects regressions section 4 disentangles the causal effect of aging from selection effects and finds that the selection effect dominates: inherent quality of the pool of surviving firms increases while the causal effect of aging on firm size is negative. Section 5 predicts the exit probability of firms and supports the selection effect: small firms are systematically more likely to exit than larger firms. Section 6 concludes.

## 2 Data

### 2.1 Administrative data in Denmark

We use three firm level data sets that are collected by Danmarks Statistik (DST), a government agency that both collects data itself as well as combines information from other government sources such as information obtained during tax collection. We combine two firm focused data sets “*Generel firmastatistik*” (FIRM) and “*Regnskabsstatistikken*” (FIRE) with additional information about employment from the worker-firm matched data set “*Beskæftigelse for lønmodtagere*” (BFL).

### 2.2 Data selection

Our data set contains the universe of Danish firms between 2001 and 2018. It contains both active and inactive firms. In general, a firms is considered active by DST if it engages

in a minimum level of economic activity (such as employing more than 0.5 full-time worker per year).

Firms in the sample that have missing information about the variables of interest could be inactive (in which case ignoring them would not be a problem), or active but not reporting. Given that smaller, less established firms are potentially more likely to not report, ignoring them would bias the results. However, given that the discrepancy between the aggregate number of employed workers reported by DST and the corresponding number in our data set is 0.36% (10k people out of 2.7M), to the degree that we are able to check, non-reporting firms seems to be inactive.

There is bunching of the founding dates in the years 1964, 1967 and 1970. These spikes are likely due to administrative reasons when the starting dates were assigned to already existing firms. We therefore remove firms from the sample that have founding dates in these years.

We use a 36 sectoral classification.<sup>1</sup> We drop firms from sectors with lots of publicly administered companies (public administration, defense, education, health care, theaters, concert halls, libraries), as well as firms that are listed as “extra-territorial bodies” and “households as employers”. For 2015, after cleaning and eliminating public sector entities, we have information about 150 thousand firms with positive employment and 0.95M million workers.

Unless stated otherwise, we only focus on firms that do not exit in the current or the next period. The reason is that younger firms are more likely to exit and we do not want the effect of aging to be mechanically driven by differences in the exit rate. However, in the appendix we report for all our analyses a direct comparison between the specification that excludes firms that exit in the next 2 years (the baseline) and a specification that uses the the full sample.

## 2.3 Variables of interest

We analyze three variables that are often used as proxies for firm size: employment, turnover and value added. While employment has been a focus of interest in the firm dynamics literature, we add value added and turnover because these variables are less likely to exhibit lumpy behavior for smaller firms.

Information about employment is collected via the tax system and it is based on com-

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<sup>1</sup>The details available here: <https://www.dst.dk/pubfile/22257/appendix>.

pulsory contributions that every worker (subject to residency registration and minimum annual earnings of roughly 1300 EUR) in Denmark makes to the labor market supplementary pension fund (ATP). Two measures of employment are utilized. For most of the analyses, similar to what is done in the literature, the employment variable (*Ansatte*) captures the headcount at the end of November, subject to a minimum activity threshold.<sup>2</sup> For analyses where we need to categorize firms into percentiles in the employment distribution, however, it turns out that this measure is not suitable. The reason is that our data set contains all firms, including the very small. A large fraction of firms in our data thus has only one or two workers. To be able to meaningfully construct size bins we therefore use full-time equivalent employment (*aatsv*) as measure of employment. For this, only the primary employment is considered for workers with multiple jobs.<sup>3</sup>

Value added (*Værditilvækst*) and turnover (*Omsætning*) comes from Accounting statistics (*Regnskabsstatistik*), which is created by DST by combining its own survey and data from SKAT and the Danish Business Authority (DBA, *Erhvervsstyrelsen*), which is an agency under the Ministry of Business Affairs. The survey is more comprehensive and DST checks the answers for internal consistency (and follows up with firms if necessary). The disadvantage is that it contains only a random sample of firms where the probability of being included is a function of firm’s size. The data from SKAT and DBA is used for taxes so should be reliable, but fewer variables are available. Both value added and turnover are measured in thousands of Danish kroner. Table 1 shows descriptive statistics of our variables of interest.

When discussing the effects of age on variable  $z$ , we present the results for levels, logs, and for growth rates. The data contains a substantial fraction of firms that do not report any employment. Most of these firms also do not report any turnover nor value added. However, occasionally they do and for that reason we do not drop them. Instead, we report both levels and logs of variables, where for the latter we essentially treat zero observations as missing. When analyzing growth rates we report results for normalized growth rates, as suggested by Haltiwanger et al. (2013):

$$\hat{g}_{z_{i,t}} \equiv \frac{z_{i,t} - z_{i,t-1}}{\frac{1}{2}(z_{i,t} + z_{i,t-1})}, \quad (1)$$

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<sup>2</sup>Workers are counted if they worked at least 80 hours in the given year, were not registered as fully unemployed in the last week of November and are residents of Denmark.

<sup>3</sup>For both measures, the primary source of information are the FIRM and FIRE registers. If the information is missing in those, we construct and use a measure of headcount from BFL register and use it instead.

Table 1: Summary of variable of interest

	N positive obs	mean	median	sdev	skewness	zeros	missing
emp nov	2.41M	6.86	3	8.93	2.13	8.77M	0.33M
emp fte	2.36M	6.37	1.9	15.24	5.09	0M	9.14M
va	1.8M	3929.84	849	11101.6	5.51	0M	9.71M
turn	4.83M	4307.68	427	14711.89	5.95	0M	6.67M

*Note:* The table reports descriptive statistics for employment (head count in November (emp nov) and full-time equivalent (emp fte), value added (va) and turnover (turn). Turnover and value added are measured in thousands of Danish kroner. The reported moments are computed excluding zeros and missing values. The non-employer firms (zero employment) constitute a disproportionately large fraction of “Agriculture”, “Other” and “Unclassified” sectors.

where  $i$  is the firm index and  $t$  is the time index.

## 2.4 Old firms in the data

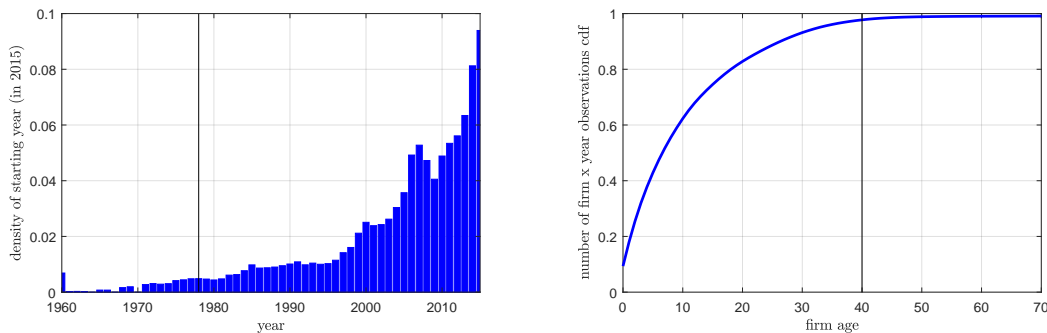
Our data set not only contains firms of all sectors and sizes. It also contains direct information on the founding date of each firm. This allows us to analyze the effects of age not only for firms that start within the sample period but for all firms, including the very old. This is in contrast to other data sets that are typically used to investigate firm dynamics. The Longitudinal Business Dataset (LBD), for example, is a confidential data set based on the Business register of the US Census and covers in its current release the period between 1976 and 2016. It contains only employer firms/establishments and does not contain information about the founding date. In that data set, firm age is therefore only known for firms that start within the sample period 1976–2016. This implies that using the LBD, effects of age can only be analyzed up until age 40.<sup>4</sup>

These old firms are a substantial fraction of the population of firms. To illustrate, figure 1, panel (a), displays the distribution of firms across founding years for all active firms in the year 2015. It shows that roughly 10% of all firms that were active in 2015 started before 1976, the earliest observable founding date in the typically used LBD. Moreover, there is a very long right tail of firm age which has been truncated in the graph to facilitate

<sup>4</sup>A well used public alternative is the Business Dynamics Statistics (BDS). BDS that is based on LBD covers the period between 1978 and 2018. However, it only provides aggregated information so individual firms cannot be traced. For example, it is possible to learn how many firms entered in 1980 and how many workers these firms employed, but it is not possible to check how many workers these firms have 10 years later, because the employment of firms aged 6-10 is reported together. In the currently available release (2018), the final age bin is 26-40 (and then all the left-censored firms together).

Figure 1: The distribution of entry year and age

- (a) Distribution of entry years (snapshot from year 2015) (b) Cumulative distribution of age (all firm-year observations)



*Note:* The figure displays the distribution of starting years and age in the sample. Panel (a) plots the distribution of starting years for all firms active in 2015. For expositional purposes, the data is left-censored at year 1960 (approximately 0.7% of firms in 2015 started in or before 1960). Panel (b) plots the cumulative distribution function of all firm-year observations by age. Vertical lines shows the cutoff where information would be truncated in the LBD data set.

readability: 1.2% of all active firms in 2015 have starting dates prior to 1960. Figure 1, panel (b), emphasizes the importance of old firms in the full sample: It displays the cumulative distribution function of all firm-year observations across firm age. It shows that more than 3% of all firm-year observations fall beyond age 40, i.e. the maximum age that is observable in the LBD data set.

In terms of employment the old firms are even more important: In 2015, firms founded before 1976 employed 10% of workers. Moreover, across all firm-year observations firms older than age 40 employed 5.4% of all workers. Our data set includes all these firms and is hence able capture this important share of economic activity.

### 3 General picture of old firms: averages over age

What do the old firms look like? In this section we document the average size of firms across age. Specifically, we compute the average age profiles of employment, turnover and value added. To eliminate the effect of short term fluctuations and sectoral differences (both in terms of levels but also in the composition of the pool of all firms), we regress the firm variables on firm age with sector and year fixed effects and we cluster the errors on



the firm level:

$$y_{ist} = \sum_{a=1}^A \beta_a I(\text{age}_{ist} = a) + \mu_s + \lambda_t + \varepsilon_{ist}, \quad (2)$$

where  $y_{ist}$  is the variable of interest for firm  $i$  in sector  $s$  at time  $t$ ,  $I(\text{age}_{ist} = a)$  is an indicator function that is equal to 1 if the age of firm  $i$  in sector  $s$  at time  $t$  is equal to  $a$ .  $\mu_s$  and  $\lambda_t$  represent sector and year fixed effects, respectively. It is likely that firms that are close to exiting become smaller right before exiting. This could lead to a bias in the estimated age coefficients in case there are systematic differences in exit rates by age. To exclude this source of bias we restrict the sample to firms that survive for at least another 2 years.<sup>5</sup>

To present the results we construct predicted values of firm outcome  $y$  for each age by averaging out the effect of sectors and years. Given that we have over 60 age coefficients, instead of showing the regression table the results are presented in figure 2. Tables with regression coefficients are listed in appendix A.

The first row of the figure shows that older firms are on average much larger. This is particularly true for the very old firms: While the average firm size is increasing across the whole age distribution, all measures of firm size (employment, value added and turnover) increase especially strongly for firms older than 40 years. For the log of size (second row of figure 2) the results are very similar: the average log(firm size) increases drastically for the very old firms.

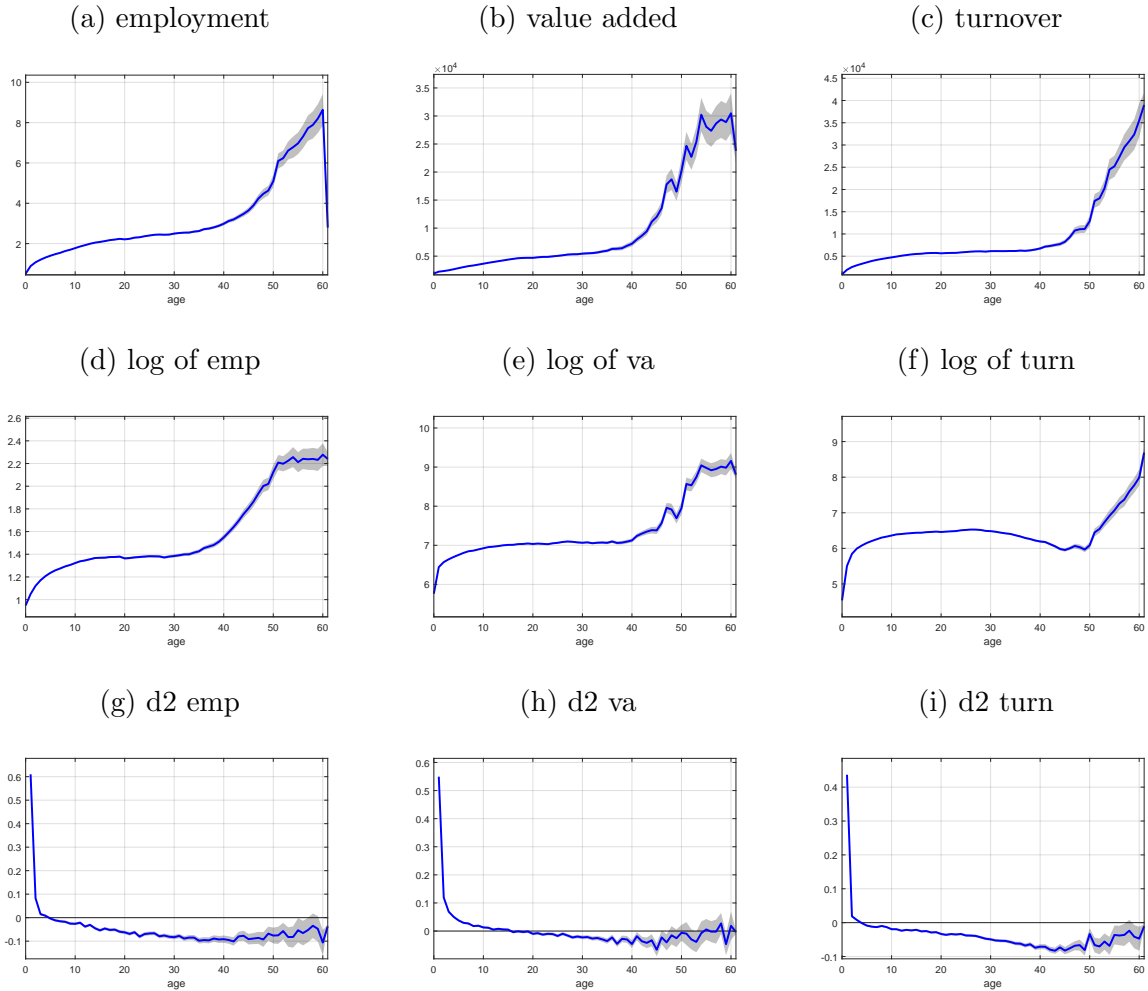
It is important to realize that this pattern does not imply that the effect of age is positive. Average firm size can increase with age for two reasons: On the one hand, it is possible that the firms become bigger as they age. This could be, for instance, because the firms gain experience in their market, their employees develop firm-specific skills, or they establish and grow their supplier and customer base, etc. On the other hand, the average firm size can also rise with age if the smallest firms are systematically more likely to exit so that only firms with increasingly high inherent quality remain at older ages. We will isolate the effects of aging from these selection effects in the next section.

The third row of figure 2 displays the average growth rate of the firms across the age distribution. We see that growth rates, in contrast to levels and logs of size, are on average decreasing over the life of a firm. We will come back to this observation in the next section

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<sup>5</sup>For comparison, appendix B.1.1 repeats the estimation on all firms, including firms that exit in the following 2 years. The estimation results are robust.

Figure 2: Age profiles



*Note:* The figure displays the predicted values from regression (2) where the outcome variables are employment (column 1), value added (column 2) and turnover (column 3), either in levels (row 1), logs (row 2) or growth rates as defined in equation (1) (row 3). Predicted values are constructed by averaging out year and sector fixed effects. Shaded areas refer to 95% confidence intervals. Value added and turnover are measured in thousands of Danish kroner.

once we have identified how aging and firm quality separately contribute to the observed average firm sizes.

## 4 Effects of aging versus selection effects

In the previous section we documented that the average firm size increases with age. In this section we now disentangle whether this is because firms become bigger as they age

or whether firm exit is non-random so that surviving firms are systematically those with higher inherent firm quality. To do that we estimate the following fixed effects model:

$$y_{ist} = \alpha_i + \sum_{a=1}^A \beta_a I(\text{age}_{ist} = a) + \mu_s + \lambda_t + \varepsilon_{ist}, \quad (3)$$

where all variables are defined as in equation (2) and  $\alpha_i$  refers to the firm fixed effect, which we interpret as inherent firm quality.<sup>6</sup>

**Effects of aging** Figure 3 shows the effects of aging once we allow for differing inherent firm quality in the form of firm fixed effects. Both the regressions in levels (row 1) and in logs (row 2) show the same patterns: Employment increases with age only for young firms. In contrast, once the firms mature the average number of employees remains approximately constant. For value added and turnover the effects of aging are even more striking: Apart from the very young firms, both value added and turnover decrease as a firm ages. This is a striking result: Even though the average firm size in the OLS regression increases with age, the pure effect of aging is negative or at best negligible for all but the very young firms.

**Selection effects** The contrast between the average firm size by age and the causal effect of aging can be explained by selection: If firm exit is non-random then the firms with higher inherent quality are more likely to survive. The pool of surviving firms therefore changes with firm age. This conclusion is supported by the change in the distribution of firm quality with age. Figure 4 shows the density function of fixed effects estimated from equation (3) for two distinct age groups of firms active in 2015: young firms (3–5 years) and old firms ( $\geq 50$  years).<sup>7</sup> In all specifications we find that the distribution of firm quality shifts to the right for older firms. This means that the pool of surviving firms is systematically different across these age groups.

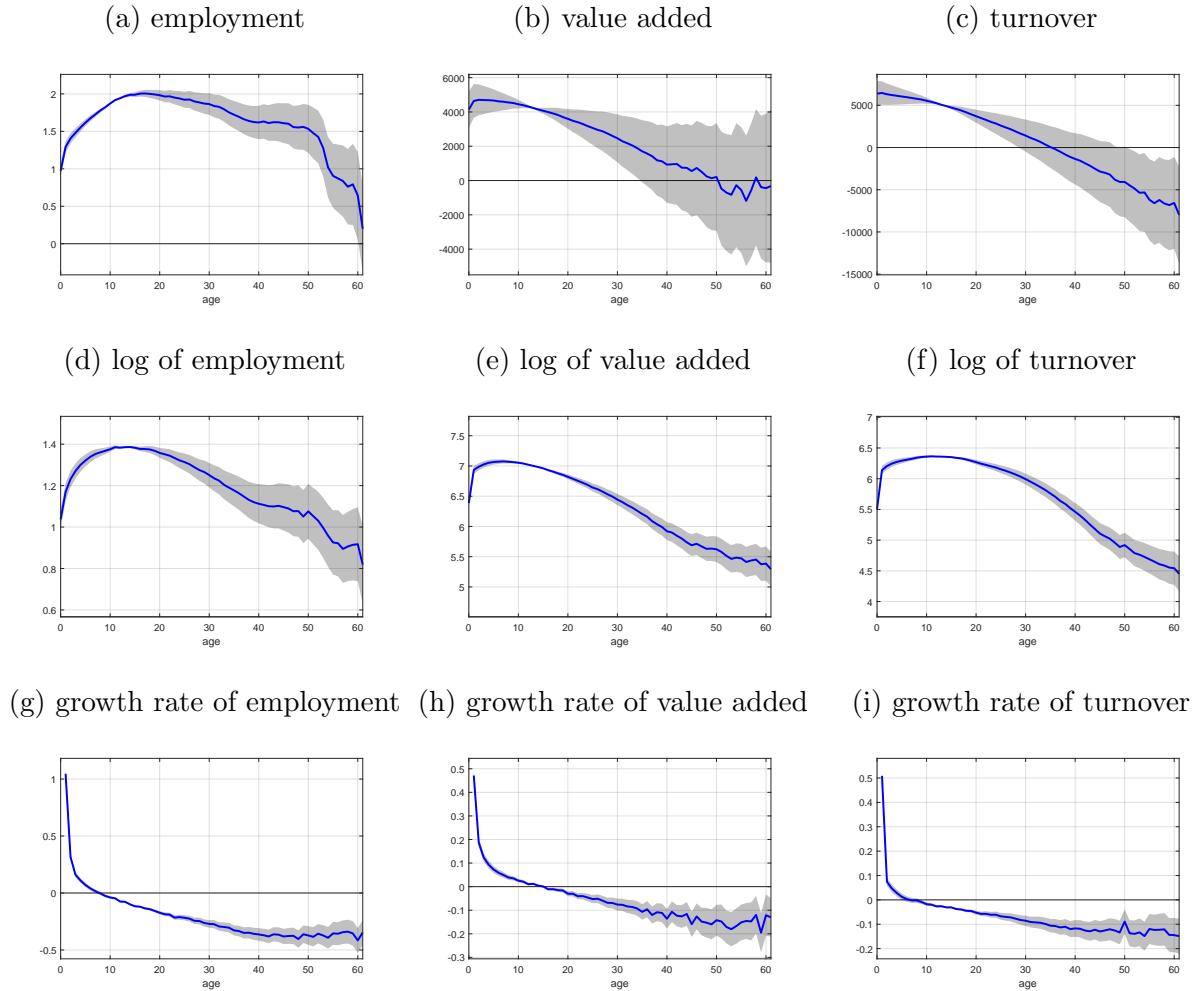
Another way of documenting the differences in the pool of surviving firms is to track the average firm quality by age. Figure 5 displays the average firm fixed effect estimated from

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<sup>6</sup>An alternative to the fixed effects model would have been a random effects model. However, the random effects model is only unbiased under the assumption that the random effects (firm quality) is uncorrelated with the regressors (age). Under the assumption that firm exit is non-random this condition would be violated, so that a random effects model cannot be justified.

<sup>7</sup>We verified in unreported robustness checks that this finding is robust to different age cutoffs and years.

Figure 3: The effect of aging

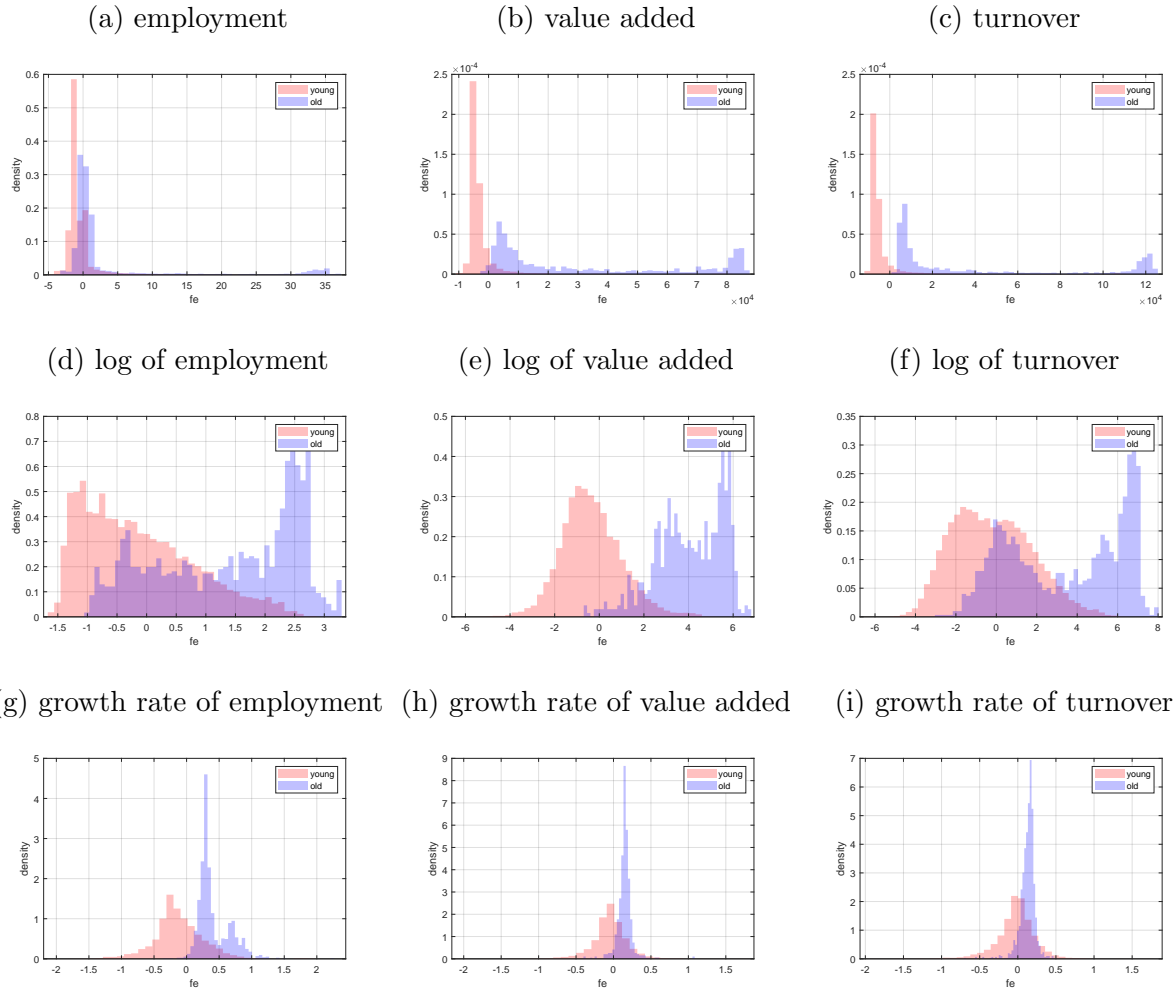


*Note:* The figure displays the predicted values from regression (3) where the outcome variables are employment (column 1), value added (column 2) and turnover (column 3), either in levels (row 1), logs (row 2) or growth rates as defined in equation (1) (row 3). Predicted values are constructed by averaging out year, sector and firm fixed effects. Shaded areas refer to 95% confidence intervals. Value added and turnover are measured in thousands of Danish kroner.

equation (3) for all firms that are active at a particular age.<sup>8</sup> Looking at the level and logs of firm size we find that the average firm quality increases with age. This is particularly pronounced for the very old firms: The age-gradient becomes much steeper around age 40. This points to a strong effect of selection at old ages: firms that survive long enough to become very old firms have an inherent firm quality that is much higher than firms that

<sup>8</sup>To interpret this figure, it is important to remember that firms are in the sample for multiple years so that each firm contributes to the averages of all ages at which it is active.

Figure 4: Densities of firm quality

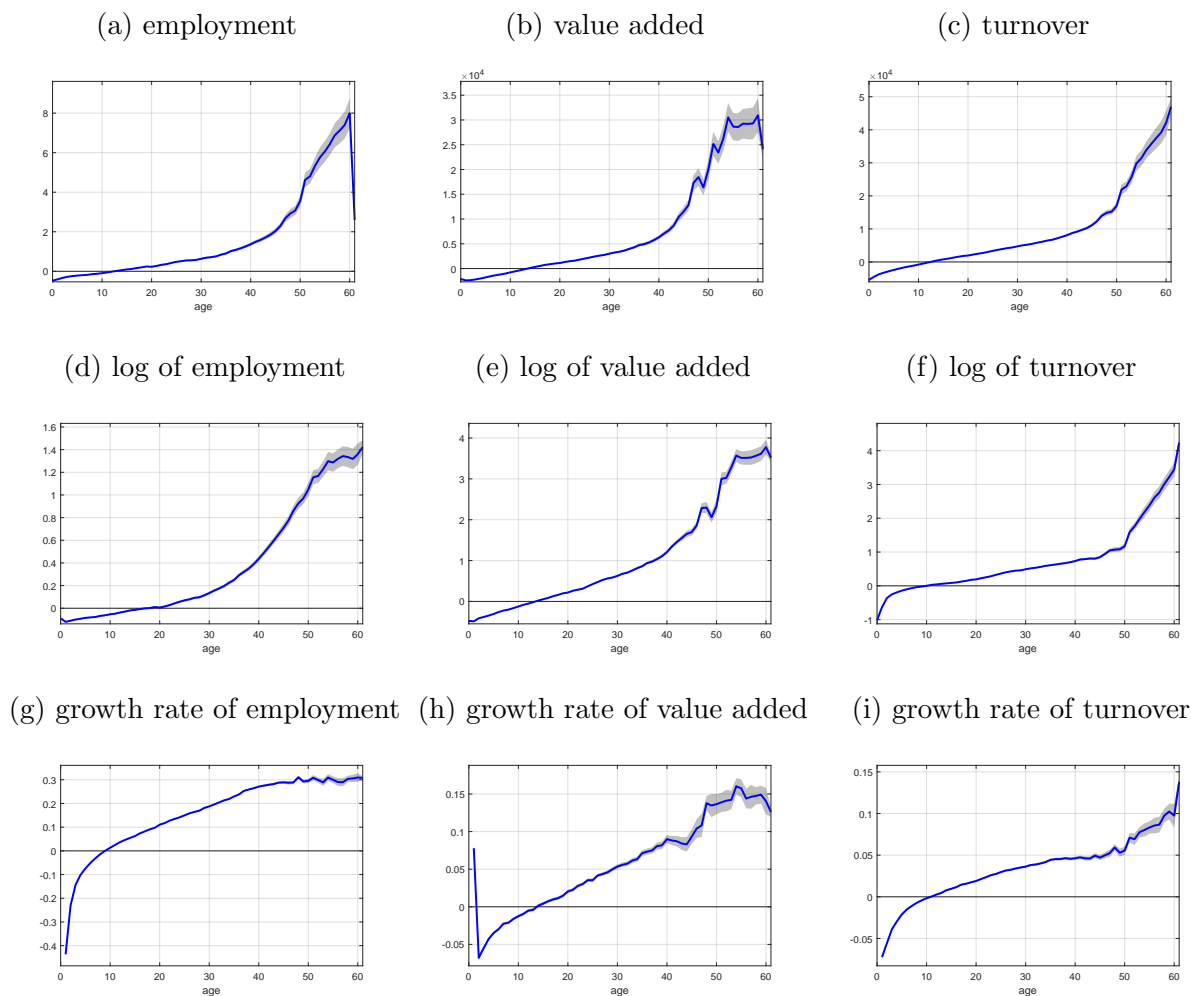


*Note:* The figure displays the density functions of the firm fixed effects estimated from equation (3) for two age groups: firms of age 3–5 years (red) and firms of 50 years and older (blue). Outcome variables are employment (column 1), value added (column 2) and turnover (column 3), either in levels (row 1), logs (row 2) or growth rates as defined in equation (1) (row 3). Value added and turnover are measured in thousands of Danish kroner.

exit prior to age 40.

**Firm growth rates** The results from the OLS regression of equation (2) showed that firm growth rates decline with age on average. This is in contrast to the results for levels or logs where we documented that average firm size increases with firm age. These contrasting results can be explained in the light of selection effects: Analyzing growth rates is similar to estimating the model in first differences. Firm inherent quality, which is constant over

Figure 5: Average firm quality by age



*Note:* The figure displays the average fixed effects estimated from regression (3) where the outcome variables are employment (column 1), value added (column 2) and turnover (column 3), either in levels (row 1), logs (row 2) or growth rates as defined in equation (1) (row 3). Shaded areas refer to 95% confidence intervals. Value added and turnover are measured in thousands of Danish kroner.

time and age for a particular firm, is therefore already differenced out. This implies that the average growth rate by age is less diluted by the selection effects present in the other specifications and we therefore observe decreasing growth rates by age on average.

But are there also firm-inherent differences in the rate of firm growth that are systematically related to the likelihood of survival? The analyses in this section show that this is indeed the case: Firms that survive longer grow on average at a higher rate across their life-cycle. This documented by the observation that the distribution of firm fixed effects in

growth rates shifts to the right for older ages (last row in figure (4)) and that the average firm fixed effect in growth rates increases with age (last row in figure (5)). This results in the observation that the net effect of aging (last row in figure (3)) is more negative than the average growth rate by age. Note, however, that in terms of growth rates the selection effect is less pronounced so that the qualitative pattern is similar between the net age effect and the average growth rate by age.

To summarize, in this section we have shown that older firms are on average larger not because they get larger with increasing age but because older firms are on average of higher quality. This is particularly true for the very old firms: They have on average substantially higher firm quality which dominates the negative effect of aging.

## 5 Mechanism: non-random firm exit

In the previous section we found that firms that are older are larger because of their higher inherent firm quality, not because aging would make firms better per se. We have argued that this finding is explained by non-random firm exit. In this section we directly test this hypothesis by examining the heterogeneity in exit rates.

We want to analyze whether exit rates are systematically different for firms in different parts of the firm size distribution. To do so we estimate the exit probability of a firm using the following logit model:

$$Pr(exit_{ist} = 1) = F \left( \alpha + \sum_{y=1}^Y \sum_{a=1}^A \beta_{ya} I(age_{ist} = a) I(size_{ist} = y) + \mu_s + \lambda_t \right), \quad (4)$$

where  $Pr(exit_{ist} = 1)$  is the probability that firm  $i$  in sector  $s$  is no longer active in the year following  $t$ .<sup>9</sup>  $I(age_{ist} = a)$  and  $I(size_{ist} = y)$  are indicator functions that are equal to 1 if the firm belongs to age group  $a$  or size percentile  $y$ , respectively.<sup>10</sup> As before we control for sector and time fixed effects.

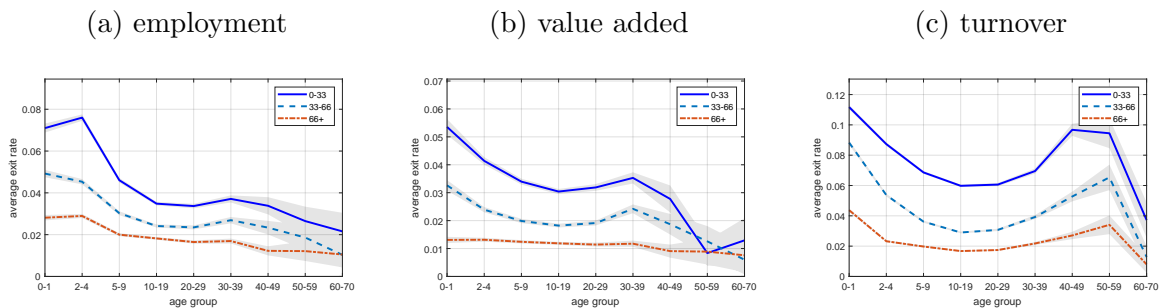
To construct the groups in the size distribution we categorize each firm  $\times$  year observation into a corresponding third of the size distribution based on the variable of interest (employment, value added, and turnover). The groups are constructed independently for

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<sup>9</sup>In appendix ?? we present results where we extend the definition of exit to include all firms that are exiting within 2 years. The resulting exit rates increase across the board, but the ranking is mostly preserved.

<sup>10</sup>Due to numerical limitations we use 9 age bins instead of age directly. Moreover, since we cannot observe survival for firms in the last sample year we drop the last year in our data set.

Figure 6: Exit rates by position in the firm distribution



*Note:* The figure displays the predicted exit probabilities by age from equation (4) for different size quartiles. Exit is defined as exiting in the current of the following two years. Panel (a) defines size quartiles based on (full-time equivalent) employment, panel (b) based on value added, and panel (c) based on turnover. The three categories characterize the exit rate from different terciles of the size distribution given by the respective variable.

all age $\times$ sector combination. The model thus estimates the likelihood that a firm from a particular segment of the distribution exits at a given age, conditional on surviving until that age. Figure 6 shows the resulting probabilities.<sup>11</sup>

We find that the probability of firm exit is indeed systematically different for firms in different parts of the size distribution: Smaller firms are more likely to exit than larger firms and this relationship holds for all age groups and definitions of size. In terms of magnitude, a firm in the smallest third of the size distribution is 2-4 times more likely to exit than a firm in the largest size third (depending on a particular age bin). This confirms that firm exit is non-random: smaller firms exit more often and hence the pool of surviving firms consists of firms that are of higher average quality (as measured by firm size).

## 6 Conclusion

In this paper we present new evidence on the characteristics of old firms. We track firm distributions over age and, perhaps not surprisingly, we document that older firms are on average larger in terms of employment, value added and turnover. The main contribution of this paper is to disentangle the causal effect of aging from the effect of selection. We

<sup>11</sup>The differences in the overall levels of exit rates across variables is due to non-reporting firms. As reported in table 1, there are differences in the number of firms that are reporting positive values for the different size variables. Given that reporting is done ex post, one possible reason for not reporting is that a firm is in liquidation: Firms in their final year are less likely to report data. For this reason, we extend the definition of exit and we focus on firms exiting in the current year or in the next two years.



find evidence for a strong selection effect: small firms are systematically more likely to exit than larger firms. This implies that the average inherent firm quality in the pool of surviving firms increases with age. At the same time, however, the causal effect of aging on firm size (measured by employment, turnover, or value added) is negative once firms mature around the age of 10 years.

This finding can have policy implications; if the population of established firms is on average better than the population of entrants, then it might be worth to target support to the established firms against the young. However, for firms of the same size, it might be better to save a young firm rather than an old firm because the young one is likely to get larger over time. Indeed, in the response to the COVID-19 recession, policy makers around the world considered various policies to support firms that have different short and long run benefits. The trade-off between saving young and small but growing firms and larger but older firms is not obvious and to fully appreciate it one has to take the growth patterns of firms into account.

Another implication of our findings concerns the quantitative theory literature. Workhorse macro models of firms typically abstract from life cycle patterns. In richer firm dynamics models, such as [Bilbiie et al. \(2012\)](#); [Clementi and Palazzo \(2016\)](#), endogenous exit occurs when some exogenous persistent process driving profits falls so much that future discounted profits are negative. Firm age, however, does not matter except for some financial friction that young firms eventually grow out of. Modeling the firm life cycle is more common in finance. [Mueller \(1972\)](#) proposed a theory that firms follow a S-shaped growth pattern of slow growth at the beginning, high growth at maturity and then an eventual slowdown due to losing their competitive advantage. This pattern has implications for dividend choices that can be tested and are supported by empirical evidence ([Fama and French, 2001](#); [DeAngelo et al., 2006](#)). In the light of our findings it seems crucial to model the micro foundation of both non-random exit and the negative effect of aging past maturity.

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## A Regression tables

Table 2: Age coefficients from average employment regression

	(1)	(2)	(3)	(4)	(5)	(6)
	emp_nov_sur	emp_nov_log_sur	d2_emp_nov_sur	emp_nov_sur	emp_nov_log_sur	d2_emp_nov_sur
age=0	0.00	(.)	0.00	(.)	0.00	(.)
age=1	0.38***	(129.0)	0.10***	(39.4)	0.32***	(69.1)
age=2	0.57***	(141.1)	0.17***	(57.9)	0.43***	(52.3)
age=3	0.69***	(145.0)	0.22***	(67.9)	0.51***	(42.2)
age=4	0.80***	(146.4)	0.26***	(74.3)	0.58***	(36.5)
age=5	0.89***	(145.4)	0.29***	(77.9)	0.64***	(32.7)
age=6	0.97***	(144.1)	0.31***	(80.2)	0.70***	(29.9)
age=7	1.04***	(141.8)	0.33***	(81.2)	0.75***	(27.6)
age=8	1.13***	(140.8)	0.34***	(82.5)	0.80***	(25.8)
age=9	1.19***	(137.5)	0.36***	(82.5)	0.85***	(24.2)
age=10	1.27***	(135.8)	0.37***	(83.4)	0.90***	(23.2)
age=11	1.35***	(133.4)	0.39***	(83.9)	0.94***	(22.1)
age=12	1.42***	(128.9)	0.40***	(82.5)	0.97***	(20.8)
age=13	1.49***	(124.5)	0.41***	(82.0)	1.00***	(19.8)
age=14	1.54***	(121.1)	0.42***	(81.5)	1.01***	(18.7)
age=15	1.58***	(117.6)	0.42***	(79.7)	1.01***	(17.5)
age=16	1.62***	(115.4)	0.42***	(78.1)	1.03***	(16.6)
age=17	1.67***	(113.2)	0.43***	(77.1)	1.03***	(15.7)
age=18	1.69***	(109.8)	0.43***	(75.0)	1.03***	(14.7)
age=19	1.73***	(106.9)	0.43***	(73.8)	1.02***	(13.9)
age=20	1.70***	(101.7)	0.42***	(69.3)	1.01***	(13.0)
age=21	1.73***	(99.1)	0.42***	(68.2)	0.99***	(12.2)
age=22	1.79***	(98.2)	0.42***	(67.9)	0.99***	(11.7)
age=23	1.81***	(96.3)	0.43***	(67.3)	0.98***	(11.0)
age=24	1.86***	(96.0)	0.43***	(66.8)	0.97***	(10.4)
age=25	1.90***	(95.1)	0.43***	(66.5)	0.95***	(9.8)
age=26	1.93***	(93.9)	0.43***	(65.2)	0.95***	(9.5)
age=27	1.94***	(91.6)	0.43***	(63.5)	0.93***	(8.9)
age=28	1.93***	(88.3)	0.42***	(60.2)	0.91***	(8.4)
age=29	1.94***	(85.8)	0.43***	(59.6)	0.90***	(8.0)
age=30	1.99***	(84.0)	0.44***	(58.3)	0.89***	(7.7)
age=31	2.02***	(80.5)	0.44***	(56.1)	0.86***	(7.2)
age=32	2.04***	(78.2)	0.45***	(55.1)	0.85***	(6.9)
age=33	2.04***	(75.2)	0.45***	(53.0)	0.82***	(6.4)
age=34	2.09***	(71.1)	0.47***	(51.3)	0.78***	(5.9)
age=35	2.12***	(68.2)	0.48***	(49.7)	0.75***	(5.5)
age=36	2.21***	(66.5)	0.51***	(50.2)	0.72***	(5.2)
age=37	2.24***	(62.9)	0.52***	(48.0)	0.69***	(4.8)
age=38	2.30***	(60.3)	0.53***	(46.2)	0.66***	(4.5)
age=39	2.38***	(58.2)	0.56***	(45.8)	0.65***	(4.3)
age=40	2.48***	(55.8)	0.60***	(45.4)	0.64***	(4.1)
age=41	2.61***	(53.7)	0.65***	(45.3)	0.66***	(4.1)
age=42	2.69***	(50.3)	0.69***	(44.4)	0.64***	(3.9)
age=43	2.83***	(47.4)	0.75***	(43.5)	0.65***	(3.9)
age=44	2.97***	(44.4)	0.81***	(42.5)	0.65***	(3.8)
age=45	3.14***	(42.0)	0.86***	(41.2)	0.63***	(3.6)
age=46	3.39***	(40.0)	0.92***	(40.5)	0.63***	(3.5)
age=47	3.73***	(36.8)	0.99***	(39.4)	0.58***	(3.1)
age=48	3.97***	(33.0)	1.05***	(37.6)	0.58***	(3.0)
age=49	4.12***	(33.3)	1.07***	(37.7)	0.59***	(3.0)
age=50	4.58***	(30.4)	1.18***	(37.2)	0.56***	(2.7)
age=51	5.59***	(28.9)	1.26***	(36.9)	0.50*	(2.4)
age=52	5.73***	(29.2)	1.25***	(36.3)	0.45*	(2.1)
age=53	6.11***	(26.6)	1.28***	(33.1)	0.31	(1.3)
age=54	6.27***	(23.4)	1.31***	(28.9)	0.04	(0.2)
age=55	6.46***	(23.5)	1.26***	(27.7)	-0.07	(-0.3)
age=56	6.80***	(23.1)	1.29***	(28.0)	-0.10	(-0.4)
age=57	7.22***	(22.4)	1.29***	(26.8)	-0.14	(-0.5)
age=58	7.38***	(21.4)	1.29***	(25.9)	-0.21	(-0.7)
age=59	7.69***	(20.9)	1.28***	(25.6)	-0.18	(-0.6)
age=60	8.13***	(20.4)	1.33***	(25.9)	-0.33	(-1.0)
age=61	2.29***	(24.1)	1.29***	(41.1)	-0.78*	(-2.4)
Constant	-0.61***	(-44.2)	0.50***	(70.9)	1.08***	(38.4)
Observations	9879035	2321568	2286060	9879035	2321568	2286060
Adjusted R <sup>2</sup>	0.098	0.100	0.037	0.026	0.037	0.070
reg. method	OLS	OLS	OLS	FE	FE	FE

t statistics in parentheses  
 \* p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001

Note: Standard errors are clustered at firm level. Age category “age = 61” includes all firms older than 60 years.

Table 3: Age coefficients from average employment (full time equivalent) regression

	(1)	(2)	(3)	(4)	(5)	(6)
	emp_fte_sur	emp_fte_log_sur	d2_emp_fte_sur	emp_fte_sur	emp_fte_log_sur	d2_emp_fte_sur
age=0	0.00 (.)	0.00 (.)		0.00 (.)	0.00 (.)	
age=1	1.11*** (47.4)	0.56*** (113.0)	0.00 (.)	1.70*** (30.9)	0.74*** (135.6)	0.00 (.)
age=2	1.55*** (54.4)	0.68*** (123.0)	-0.52*** (-180.4)	2.26*** (21.7)	0.88*** (109.4)	-0.58*** (-191.8)
age=3	1.92*** (60.5)	0.74*** (128.9)	-0.58*** (-207.3)	2.62*** (17.0)	0.94*** (87.2)	-0.68*** (-207.4)
age=4	2.26*** (65.2)	0.80*** (133.2)	-0.60*** (-215.1)	2.96*** (14.5)	1.00*** (73.0)	-0.71*** (-198.3)
age=5	2.58*** (68.2)	0.85*** (135.3)	-0.61*** (-219.3)	3.27*** (12.8)	1.05*** (62.8)	-0.73*** (-183.9)
age=6	2.85*** (70.4)	0.87*** (135.5)	-0.62*** (-221.7)	3.54*** (11.6)	1.09*** (55.0)	-0.75*** (-170.6)
age=7	3.11*** (71.9)	0.90*** (136.1)	-0.62*** (-221.2)	3.84*** (10.8)	1.13*** (49.5)	-0.76*** (-155.6)
age=8	3.35*** (72.9)	0.93*** (136.6)	-0.62*** (-221.0)	4.10*** (10.1)	1.17*** (44.9)	-0.77*** (-142.3)
age=9	3.57*** (73.1)	0.95*** (136.0)	-0.63*** (-224.4)	4.33*** (9.5)	1.19*** (41.0)	-0.79*** (-132.6)
age=10	3.83*** (73.5)	0.97*** (135.6)	-0.63*** (-222.9)	4.56*** (9.0)	1.22*** (38.1)	-0.79*** (-122.1)
age=11	4.07*** (73.3)	0.99*** (135.3)	-0.63*** (-221.0)	4.78*** (8.6)	1.26*** (35.6)	-0.79*** (-112.9)
age=12	4.29*** (72.4)	1.00*** (133.0)	-0.64*** (-221.6)	4.99*** (8.2)	1.28*** (33.3)	-0.81*** (-105.9)
age=13	4.45*** (71.1)	1.01*** (129.2)	-0.64*** (-220.3)	5.16*** (7.8)	1.30*** (31.3)	-0.81*** (-99.3)
age=14	4.59*** (69.7)	1.02*** (128.0)	-0.64*** (-217.7)	5.30*** (7.5)	1.33*** (29.7)	-0.82*** (-93.3)
age=15	4.76*** (68.2)	1.02*** (124.6)	-0.65*** (-217.1)	5.48*** (7.2)	1.34*** (28.1)	-0.83*** (-88.6)
age=16	4.91*** (67.3)	1.02*** (122.6)	-0.65*** (-215.9)	5.61*** (6.9)	1.36*** (26.6)	-0.83*** (-83.7)
age=17	5.02*** (65.8)	1.02*** (119.5)	-0.65*** (-217.0)	5.77*** (6.7)	1.37*** (25.3)	-0.84*** (-79.8)
age=18	5.10*** (63.9)	1.02*** (116.8)	-0.65*** (-211.5)	5.90*** (6.5)	1.39*** (24.2)	-0.84*** (-75.5)
age=19	5.16*** (62.2)	1.02*** (114.0)	-0.65*** (-210.6)	5.99*** (6.2)	1.39*** (23.0)	-0.85*** (-72.5)
age=20	5.12*** (59.0)	1.01*** (110.0)	-0.65*** (-206.4)	6.13*** (6.0)	1.41*** (22.1)	-0.86*** (-69.3)
age=21	5.22*** (58.0)	1.02*** (108.1)	-0.65*** (-203.0)	6.25*** (5.9)	1.42*** (21.2)	-0.86*** (-66.5)
age=22	5.33*** (57.4)	1.02*** (106.1)	-0.66*** (-202.0)	6.33*** (5.7)	1.42*** (20.3)	-0.87*** (-64.3)
age=23	5.40*** (57.1)	1.02*** (103.1)	-0.66*** (-200.1)	6.43*** (5.5)	1.42*** (19.5)	-0.88*** (-62.0)
age=24	5.51*** (57.1)	1.03*** (103.9)	-0.66*** (-195.9)	6.53*** (5.4)	1.44*** (18.9)	-0.88*** (-59.7)
age=25	5.54*** (56.7)	1.03*** (102.2)	-0.66*** (-195.5)	6.57*** (5.2)	1.44*** (18.1)	-0.89*** (-57.9)
age=26	5.64*** (56.2)	1.04*** (102.4)	-0.66*** (-192.8)	6.64*** (5.0)	1.46*** (17.7)	-0.89*** (-55.6)
age=27	5.68*** (55.0)	1.04*** (99.4)	-0.66*** (-194.1)	6.73*** (4.9)	1.46*** (17.0)	-0.90*** (-54.3)
age=28	5.72*** (53.5)	1.03*** (95.8)	-0.67*** (-191.0)	6.80*** (4.8)	1.46*** (16.4)	-0.91*** (-52.7)
age=29	5.85*** (52.5)	1.03*** (92.2)	-0.67*** (-186.0)	6.86*** (4.7)	1.46*** (15.9)	-0.92*** (-51.1)
age=30	6.10*** (52.2)	1.02*** (88.6)	-0.67*** (-181.4)	6.94*** (4.6)	1.45*** (15.2)	-0.92*** (-49.8)
age=31	6.18*** (50.3)	1.02*** (84.0)	-0.67*** (-177.6)	6.94*** (4.4)	1.44*** (14.7)	-0.93*** (-48.5)
age=32	6.26*** (49.0)	1.03*** (82.3)	-0.67*** (-169.3)	6.96*** (4.3)	1.45*** (14.3)	-0.93*** (-47.0)
age=33	6.36*** (47.6)	1.02*** (78.1)	-0.67*** (-167.6)	6.95*** (4.1)	1.43*** (13.7)	-0.94*** (-46.1)
age=34	6.52*** (45.5)	1.03*** (73.9)	-0.67*** (-157.9)	6.94*** (4.0)	1.42*** (13.2)	-0.95*** (-44.9)
age=35	6.57*** (43.4)	1.03*** (69.6)	-0.68*** (-149.3)	6.92*** (3.9)	1.41*** (12.7)	-0.95*** (-43.9)
age=36	6.76*** (42.2)	1.04*** (66.7)	-0.67*** (-141.9)	6.94*** (3.8)	1.40*** (12.2)	-0.95*** (-42.6)
age=37	6.90*** (40.3)	1.04*** (62.4)	-0.68*** (-137.3)	6.90*** (3.7)	1.38*** (11.8)	-0.97*** (-41.9)
age=38	7.21*** (39.0)	1.08*** (60.8)	-0.67*** (-129.1)	6.87*** (3.6)	1.38*** (11.5)	-0.96*** (-40.6)
age=39	7.59*** (37.7)	1.08*** (56.8)	-0.68*** (-123.9)	6.92*** (3.5)	1.36*** (11.0)	-0.98*** (-40.2)
age=40	8.16*** (36.3)	1.12*** (53.6)	-0.67*** (-114.9)	6.89*** (3.4)	1.36*** (10.7)	-0.97*** (-38.6)
age=41	8.87*** (35.0)	1.20*** (54.9)	-0.66*** (-107.6)	7.05*** (3.4)	1.38*** (10.6)	-0.96*** (-37.3)
age=42	9.50*** (33.1)	1.26*** (53.0)	-0.68*** (-105.7)	7.05*** (3.3)	1.38*** (10.3)	-0.99*** (-37.2)
age=43	10.37*** (31.3)	1.31*** (49.4)	-0.67*** (-93.0)	7.08*** (3.2)	1.37*** (10.0)	-0.98*** (-35.9)
age=44	11.73*** (29.5)	1.38*** (46.3)	-0.66*** (-88.0)	7.09*** (3.2)	1.36*** (9.7)	-0.97*** (-34.8)
age=45	13.09*** (28.3)	1.45*** (43.5)	-0.68*** (-83.3)	7.02*** (3.1)	1.36*** (9.5)	-1.00*** (-34.7)
age=46	14.17*** (26.8)	1.54*** (42.4)	-0.66*** (-77.3)	7.06*** (3.0)	1.38*** (9.4)	-0.98*** (-33.3)
age=47	15.82*** (25.2)	1.66*** (40.4)	-0.67*** (-72.8)	6.76*** (2.8)	1.37*** (9.1)	-0.99*** (-32.8)
age=48	16.83*** (22.5)	1.79*** (40.4)	-0.64*** (-63.0)	6.74*** (2.8)	1.45*** (9.5)	-0.97*** (-31.2)
age=49	16.95*** (22.4)	1.74*** (37.8)	-0.66*** (-66.7)	6.74*** (2.7)	1.45*** (9.3)	-1.00*** (-31.4)
age=50	19.23*** (20.9)	1.91*** (37.0)	-0.65*** (-64.7)	6.58*** (2.6)	1.50*** (9.3)	-0.99*** (-30.6)
age=51	22.25*** (20.7)	2.14*** (40.2)	-0.65*** (-56.2)	6.44*** (2.5)	1.54*** (9.4)	-0.99*** (-29.5)
age=52	21.59*** (20.5)	2.08*** (37.0)	-0.69*** (-55.9)	5.71*** (2.1)	1.46*** (8.7)	-1.03*** (-29.9)
age=53	23.45*** (19.6)	2.11*** (33.5)	-0.71*** (-50.2)	5.35*** (1.9)	1.43*** (8.4)	-1.05*** (-29.3)
age=54	28.56*** (19.0)	2.21*** (27.4)	-0.66*** (-40.8)	4.73*** (1.7)	1.41*** (8.0)	-1.01*** (-27.1)
age=55	28.36*** (19.1)	2.22*** (28.1)	-0.66*** (-47.1)	5.24*** (1.8)	1.43*** (8.0)	-1.02*** (-27.5)
age=56	27.94*** (18.8)	2.17*** (27.1)	-0.65*** (-42.2)	4.63*** (1.6)	1.43*** (7.8)	-0.99*** (-26.2)
age=57	28.67*** (18.2)	2.24*** (27.7)	-0.65*** (-39.3)	4.34*** (1.4)	1.47*** (7.9)	-1.00*** (-25.4)
age=58	28.54*** (17.3)	2.23*** (25.1)	-0.69*** (-37.2)	3.87*** (1.3)	1.44*** (7.6)	-1.02*** (-25.4)
age=59	29.09*** (17.0)	2.27*** (26.6)	-0.60*** (-34.7)	4.43*** (1.4)	1.54*** (7.9)	-0.95*** (-24.4)
age=60	29.77*** (16.7)	2.34*** (27.3)	-0.64*** (-33.1)	4.62*** (1.4)	1.59*** (8.0)	-0.99*** (-23.5)
age=61	23.08*** (22.3)	1.82*** (35.2)	-0.66*** (-183.3)	3.14*** (0.9)	1.54*** (7.3)	-1.01*** (-24.6)
Constant	-0.51*** (-6.9)	-0.36*** (-35.8)	0.63*** (224.8)	2.61*** (8.9)	-0.21*** (-5.7)	0.76*** (55.0)
Observations	2250870	2250870	2091786	2250870	2250870	2091786
Adjusted R <sup>2</sup>	0.097	0.134	0.078	0.028	0.062	0.086
reg. method	OLS	OLS	OLS	FE	FE	FE

*t* statistics in parentheses  
\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Note: Standard errors are clustered at firm level. Age category “age = 61” includes all firms older than 60 years.



Table 5: Age coefficients from average turnover regression

	(1)	(2)	(3)	(4)	(5)	(6)
	turn_sur	turn_log_sur	d2_turn_sur	turn_sur	turn_log_sur	d2_turn_sur
age=0	0.00	0.00		0.00	0.00	
age=1	1014.37*** (63.2)	0.97*** (235.0)	0.00	81.37 (1.4)	0.63*** (146.5)	0.00
age=2	1632.72*** (82.2)	1.31*** (275.0)	-0.42*** (-148.3)	-70.04 (-0.6)	0.70*** (105.2)	-0.43*** (-132.4)
age=3	2027.16*** (90.2)	1.46*** (287.0)	-0.43*** (-162.5)	-165.79 (-1.0)	0.74*** (81.1)	-0.46*** (-136.8)
age=4	2362.94*** (94.6)	1.54*** (286.9)	-0.44*** (-167.3)	-250.41 (-1.1)	0.76*** (65.6)	-0.48*** (-130.1)
age=5	2680.32*** (97.8)	1.61*** (286.9)	-0.45*** (-169.4)	-339.70 (-1.2)	0.78*** (54.9)	-0.50*** (-120.2)
age=6	2990.12*** (99.8)	1.67*** (285.5)	-0.45*** (-170.5)	-419.93 (-1.2)	0.80*** (47.2)	-0.51*** (-109.7)
age=7	3236.83*** (100.6)	1.72*** (284.2)	-0.45*** (-170.8)	-511.09 (-1.3)	0.81*** (41.6)	-0.51*** (-99.6)
age=8	3442.91*** (99.9)	1.76*** (283.5)	-0.45*** (-169.1)	-608.12 (-1.3)	0.83*** (37.4)	-0.51*** (-89.9)
age=9	3635.55*** (98.7)	1.79*** (279.7)	-0.45*** (-170.4)	-696.26 (-1.3)	0.84*** (33.8)	-0.52*** (-82.6)
age=10	3804.15*** (97.9)	1.82*** (277.5)	-0.46*** (-172.4)	-841.35 (-1.5)	0.85*** (30.7)	-0.53*** (-76.6)
age=11	3983.11*** (96.6)	1.85*** (274.8)	-0.46*** (-171.7)	-980.12 (-1.5)	0.86*** (28.2)	-0.53*** (-70.6)
age=12	4174.75*** (94.4)	1.87*** (267.4)	-0.46*** (-172.2)	-1132.20 (-1.6)	0.85*** (25.8)	-0.54*** (-66.0)
age=13	4321.97*** (92.6)	1.88*** (261.7)	-0.46*** (-170.7)	-1316.83 (-1.8)	0.85*** (23.8)	-0.53*** (-61.2)
age=14	4473.57*** (90.6)	1.89*** (255.5)	-0.46*** (-169.8)	-1479.35 (-1.8)	0.85*** (22.0)	-0.54*** (-57.5)
age=15	4570.75*** (88.9)	1.90*** (251.2)	-0.46*** (-168.5)	-1647.70 (-1.9)	0.84*** (20.4)	-0.54*** (-53.8)
age=16	4623.80*** (87.3)	1.90*** (247.7)	-0.46*** (-169.5)	-1818.93* (-2.0)	0.83*** (18.9)	-0.55*** (-51.2)
age=17	4745.70*** (85.9)	1.92*** (244.9)	-0.46*** (-168.6)	-1989.00* (-2.0)	0.82*** (17.6)	-0.55*** (-48.4)
age=18	4786.72*** (83.4)	1.92*** (240.1)	-0.47*** (-168.9)	-2213.43* (-2.1)	0.80*** (16.3)	-0.55*** (-46.2)
age=19	4800.10*** (81.0)	1.93*** (235.3)	-0.46*** (-167.8)	-2445.16* (-2.2)	0.78*** (15.0)	-0.55*** (-43.9)
age=20	4711.09*** (77.3)	1.92*** (228.4)	-0.47*** (-167.8)	-2669.32* (-2.3)	0.76*** (13.9)	-0.56*** (-42.2)
age=21	4773.39*** (75.3)	1.93*** (224.4)	-0.47*** (-167.6)	-2891.42* (-2.4)	0.74*** (12.8)	-0.57*** (-40.6)
age=22	4806.34*** (73.6)	1.94*** (221.0)	-0.47*** (-165.1)	-3111.86* (-2.4)	0.72*** (11.9)	-0.57*** (-38.7)
age=23	4818.28*** (72.2)	1.95*** (218.5)	-0.47*** (-164.7)	-3346.09* (-2.5)	0.69*** (11.0)	-0.57*** (-37.3)
age=24	4921.94*** (71.7)	1.97*** (217.9)	-0.47*** (-163.0)	-3540.80* (-2.6)	0.67*** (10.2)	-0.57*** (-35.7)
age=25	5004.12*** (70.7)	1.98*** (215.1)	-0.47*** (-163.9)	-3789.16** (-2.6)	0.64*** (9.4)	-0.58*** (-34.7)
age=26	5095.63*** (70.0)	1.99*** (212.4)	-0.47*** (-163.0)	-4006.34** (-2.7)	0.62*** (8.7)	-0.58*** (-33.5)
age=27	5150.14*** (68.2)	1.99*** (207.3)	-0.48*** (-162.0)	-4219.59** (-2.7)	0.59*** (8.0)	-0.58*** (-32.3)
age=28	5157.60*** (66.5)	1.98*** (201.6)	-0.48*** (-161.2)	-4485.23** (-2.8)	0.56*** (7.3)	-0.59*** (-31.5)
age=29	5102.00*** (64.2)	1.95*** (195.2)	-0.48*** (-160.3)	-4738.81** (-2.8)	0.52*** (6.6)	-0.59*** (-30.7)
age=30	5204.95*** (62.6)	1.95*** (189.2)	-0.49*** (-159.0)	-4968.96** (-2.9)	0.48*** (5.9)	-0.59*** (-29.8)
age=31	5215.58*** (59.6)	1.93*** (178.8)	-0.49*** (-157.7)	-5240.59** (-2.9)	0.44*** (5.2)	-0.60*** (-29.0)
age=32	5205.39*** (57.4)	1.90*** (171.7)	-0.49*** (-153.3)	-5464.35** (-3.0)	0.40*** (4.6)	-0.60*** (-28.2)
age=33	5207.30*** (55.1)	1.88*** (164.7)	-0.49*** (-149.6)	-5722.42** (-3.0)	0.35*** (3.9)	-0.60*** (-27.4)
age=34	5242.88*** (51.7)	1.87*** (153.1)	-0.49*** (-146.9)	-6035.36** (-3.1)	0.31*** (3.3)	-0.61*** (-26.8)
age=35	5236.61*** (49.1)	1.83*** (143.1)	-0.50*** (-141.4)	-6329.99** (-3.1)	0.25** (2.6)	-0.61*** (-26.3)
age=36	5329.17*** (46.9)	1.80*** (134.4)	-0.50*** (-135.4)	-6579.20** (-3.2)	0.20* (2.0)	-0.62*** (-25.6)
age=37	5275.31*** (43.6)	1.75*** (124.3)	-0.50*** (-133.0)	-6881.23** (-3.2)	0.13 (1.3)	-0.62*** (-25.1)
age=38	5383.38*** (41.7)	1.72*** (116.8)	-0.50*** (-128.2)	-7210.13** (-3.3)	0.08 (0.8)	-0.62*** (-24.3)
age=39	5563.44*** (39.9)	1.68*** (107.8)	-0.51*** (-125.0)	-7494.97** (-3.3)	0.01 (0.1)	-0.63*** (-24.0)
age=40	5810.19*** (38.4)	1.66*** (100.2)	-0.51*** (-117.1)	-7732.33** (-3.3)	-0.05 (-0.4)	-0.63*** (-23.3)
age=41	6222.39*** (36.9)	1.64*** (92.8)	-0.51*** (-112.2)	-7938.59** (-3.3)	-0.11 (-1.0)	-0.63*** (-22.8)
age=42	6363.50*** (34.6)	1.58*** (83.0)	-0.52*** (-107.4)	-8245.36** (-3.4)	-0.18 (-1.6)	-0.63*** (-22.4)
age=43	6591.12*** (32.3)	1.52*** (73.5)	-0.52*** (-100.2)	-8539.83** (-3.4)	-0.26* (-2.2)	-0.64*** (-22.0)
age=44	6790.68*** (30.0)	1.45*** (64.6)	-0.51*** (-91.1)	-8892.86** (-3.5)	-0.33** (-2.7)	-0.63*** (-21.2)
age=45	7305.81*** (28.7)	1.42*** (58.1)	-0.52*** (-86.9)	-9222.12** (-3.5)	-0.40** (-3.3)	-0.64*** (-20.9)
age=46	8368.42*** (28.0)	1.46*** (53.8)	-0.51*** (-77.3)	-9371.57** (-3.5)	-0.45*** (-3.5)	-0.63*** (-20.3)
age=47	9837.86*** (25.8)	1.53*** (46.7)	-0.50*** (-69.9)	-9580.48** (-3.5)	-0.49*** (-3.8)	-0.63*** (-19.6)
age=48	10132.88*** (22.5)	1.50*** (39.3)	-0.50*** (-58.6)	-1.0e + 04*** (-3.7)	-0.55*** (-4.2)	-0.63*** (-19.1)
age=49	10186.80*** (22.5)	1.43*** (37.8)	-0.52*** (-60.5)	-1.0e + 04*** (-3.7)	-0.62*** (-4.6)	-0.64*** (-19.0)
age=50	11927.76*** (20.7)	1.55*** (33.8)	-0.47*** (-52.3)	-1.0e + 04*** (-3.6)	-0.59*** (-4.3)	-0.60*** (-17.2)
age=51	16503.73*** (20.8)	1.91*** (32.6)	-0.50*** (-44.2)	-1.1e + 04*** (-3.7)	-0.65*** (-4.6)	-0.65*** (-17.9)
age=52	17095.15*** (21.0)	2.01*** (33.8)	-0.51*** (-42.0)	-1.1e + 04*** (-3.7)	-0.72*** (-5.0)	-0.65*** (-17.6)
age=53	19333.70*** (20.1)	2.20*** (31.7)	-0.49*** (-39.7)	-1.2e + 04*** (-3.8)	-0.75*** (-5.1)	-0.64*** (-17.0)
age=54	23547.92*** (19.8)	2.38*** (28.8)	-0.50*** (-36.6)	-1.2e + 04*** (-3.7)	-0.78*** (-5.2)	-0.66*** (-17.0)
age=55	24280.75*** (19.8)	2.53*** (30.0)	-0.47*** (-33.0)	-1.3e + 04*** (-3.9)	-0.82*** (-5.4)	-0.63*** (-15.7)
age=56	26341.06*** (19.7)	2.72*** (30.3)	-0.47*** (-31.9)	-1.3e + 04*** (-4.0)	-0.86*** (-5.5)	-0.63*** (-15.6)
age=57	28557.04*** (19.0)	2.84*** (28.7)	-0.47*** (-28.2)	-1.3e + 04*** (-3.8)	-0.90*** (-5.6)	-0.63*** (-15.1)
age=58	29937.42*** (18.1)	3.06*** (29.5)	-0.46*** (-27.4)	-1.3e + 04*** (-3.8)	-0.92*** (-5.6)	-0.63*** (-14.9)
age=59	31440.16*** (17.9)	3.24*** (30.1)	-0.48*** (-26.9)	-1.3e + 04*** (-3.8)	-0.96*** (-5.7)	-0.65*** (-15.0)
age=60	34595.87*** (17.8)	3.45*** (29.6)	-0.48*** (-26.5)	-1.3e + 04*** (-3.7)	-0.97*** (-5.6)	-0.65*** (-14.8)
age=61	38074.64*** (27.7)	4.15*** (60.0)	-0.45*** (-119.4)	-1.4e + 04*** (-3.9)	-1.06*** (-5.9)	-0.66*** (-14.6)
Constant	-2817.73*** (-62.7)	3.98*** (463.5)	0.43*** (167.2)	2969.65*** (12.3)	5.21*** (254.1)	0.48*** (73.5)
Observations	4451548	4451548	3370743	4451548	4451548	3370743
Adjusted R <sup>2</sup>	0.096	0.154	0.037	0.014	0.061	0.035
reg. method	OLS	OLS	OLS	FE	FE	FE

t statistics in parentheses  
\* p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001

Note: Standard errors are clustered at firm level. Age category “age = 31” includes all firms older than 60 years.

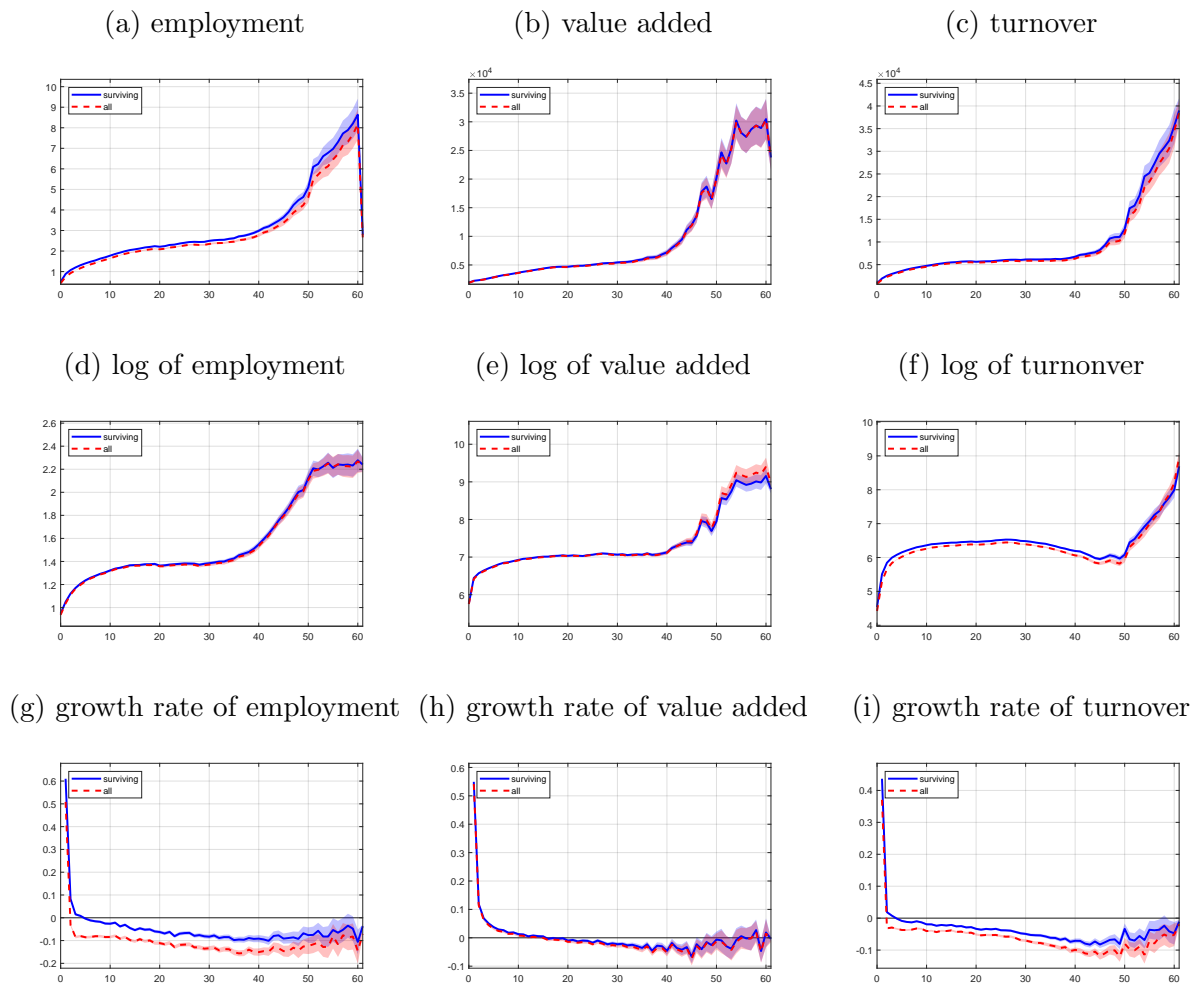


## B Robustness

### B.1 All firms vs survivors

#### B.1.1 Age profiles

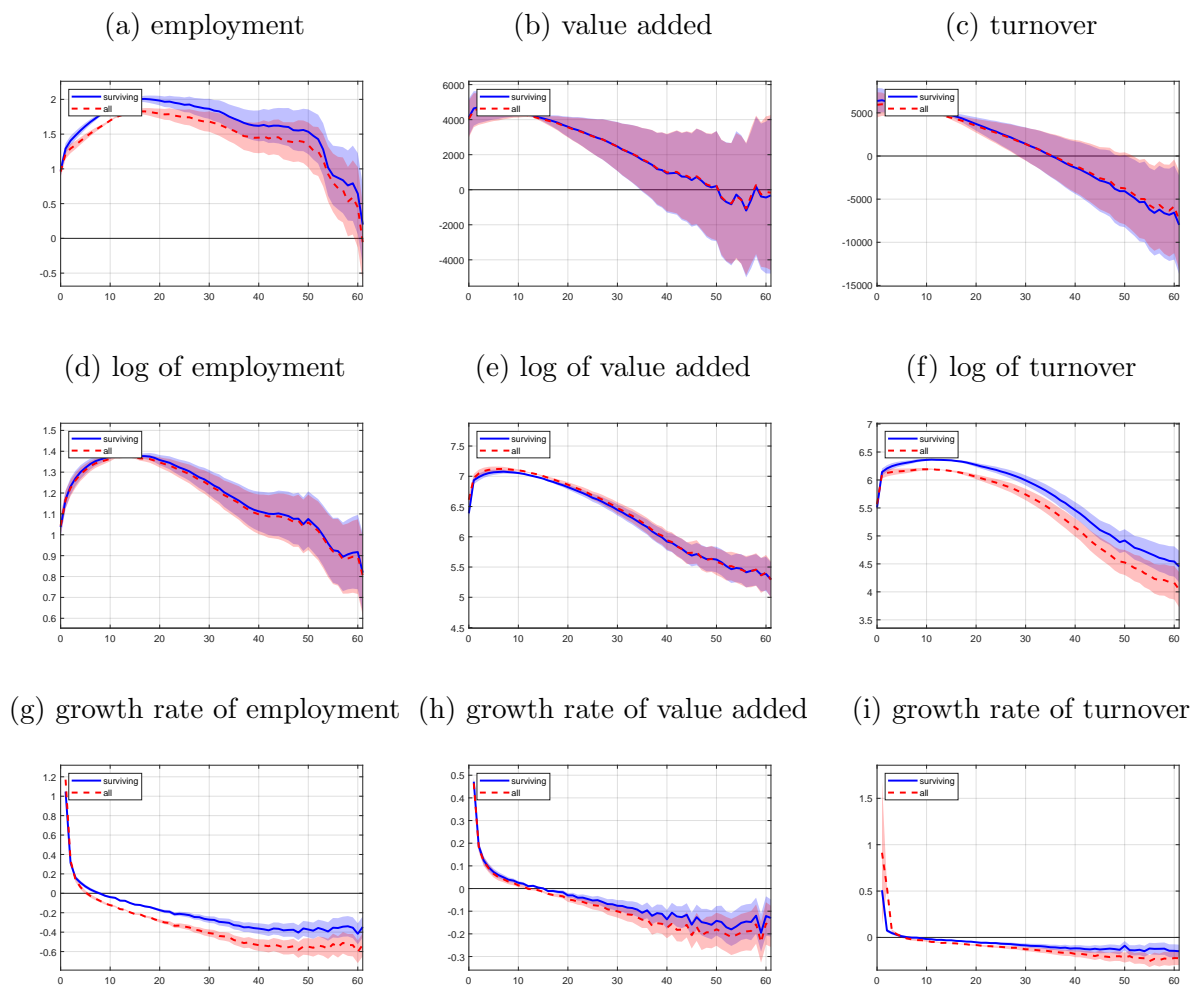
Figure 7: Surviving firms vs all firms



*Note:* The figure displays the average size estimated on two different samples: all firms (red line) and firms that survive for at least another 2 years (blue line). Estimates based on regression (2) where the outcome variables are employment (column 1), value added (column 2) and turnover (column 3), either in levels (row 1), logs (row 2) or growth rates as defined in equation (1) (row 3). Shaded areas refer to 95% confidence intervals. Value added and turnover are measured in thousands of Danish kroner.

## B.1.2 Age effects

Figure 8: Surviving firms vs all firms



*Note:* The figure displays the predicted age effects estimated on two different samples: all firms (red line) and firms that survive for at least another 2 years (blue line). Estimates based on regression (3) where the outcome variables are employment (column 1), value added (column 2) and turnover (column 3), either in levels (row 1), logs (row 2) or growth rates as defined in equation (1) (row 3). Predicted values are constructed by averaging out year, sector and firm fixed effects. Shaded areas refer to 95% confidence intervals. Value added and turnover are measured in thousands of Danish kroner.

## C Estimating the selection issues

The econometric challenge is how to disentangle the effect of aging from the firm-specific quality that is constant over the firm life subject when the length of the data set is shorter than the potential firm lifespan. The answer is to exploit the overlapping structure: firms that are observed over their years 0 and  $T$  can be used to help estimate firms that are present between ages 1 and  $T + 1$  and so on. This section contains the background on the identification issues in this framework.

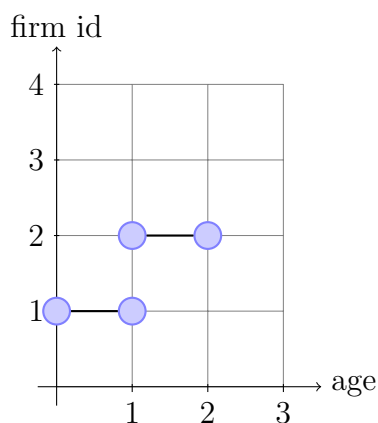
Suppose that the outcome  $x$  of firm  $i$  at age  $a$  is given by

$$x_{i,a} = \alpha_i + \gamma_a$$

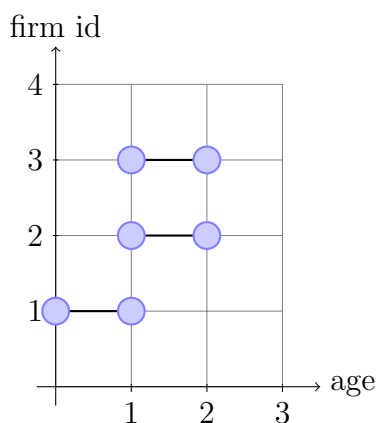
and that the data set consists of  $N$  firms that are observed over  $M$  ages, but not all firms are observed at all ages, as depicted in figure 9. The question is: under what conditions are the firm-specific fixed effects  $\alpha_i$  and the common age effects  $\gamma$  identified? The total number of coefficients is  $N + M$  (one for each firm and one for each age).

Figure 9: Two examples when the identification fails

(a) Example 1: an example with is not enough data to separately identify age and firm effects



(b) Example 2: age and firm fixed effects not identified even though as many data points as coefficients



Trivially, if you have fewer firm  $\times$  age observations than coefficients, the model is not identified. As in the example depicted figure 9(a), there are 2 firms observed each for two years with one year overlapping. In this example, there are 4 different observations

for 5 coefficients ( $\alpha_1, \alpha_2, \gamma_1, \gamma_2, \gamma_3$ ) and the coefficients cannot be identified. However, it is not enough to only have the number of observations at least as high as the number of coefficients. In the example in figure 9(b), there are 6 coefficients ( $\alpha_1, \alpha_2, \alpha_3, \gamma_1, \gamma_2, \gamma_3$ ) and 6 data points and yet, the coefficients cannot be identified. The easiest way to see this is to construct the system in matrix form:

$$\begin{bmatrix} x_{1,1} \\ x_{1,2} \\ x_{2,2} \\ x_{2,3} \\ x_{3,2} \\ x_{3,3} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{B}} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix}$$

and the rank of matrix  $\mathbf{B}$  is only 5, so the system does not have a unique solution and hence the coefficients cannot be identified.

An example of identified system is depicted in figure 10.

In this example,

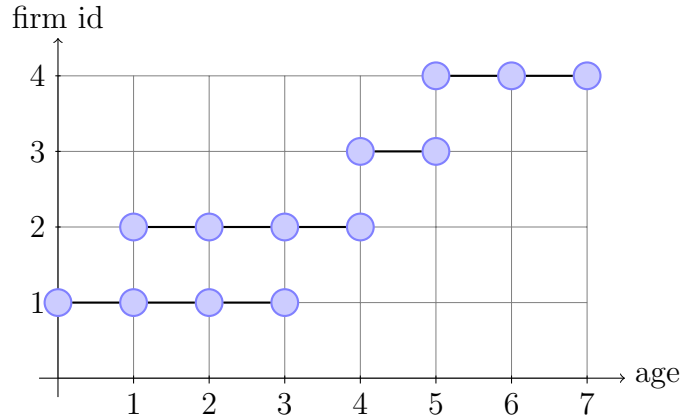
1.  $x_{1,1}, x_{1,2}, x_{1,3}, x_{2,1}, x_{2,2}, x_{2,3}$  uniquely identify  $\alpha_1, \alpha_2, \gamma_1, \gamma_2, \gamma_3$ .
2. with  $\alpha_1$  known,  $x_{1,0}$  identifies  $\gamma_0$
3. with now  $\alpha_2$  known,  $x_{2,4}$  identifies  $\gamma_4$
4. with now  $\gamma_4$  known,  $x_{3,4}$  identifies  $\alpha_3$
5. with now  $\alpha_3$  known,  $x_{3,5}$  identifies  $\gamma_5$
6. with now  $\gamma_5$  known,  $x_{4,5}$  identifies  $\alpha_4$
7. with now  $\alpha_4$  known,  $x_{4,6}, x_{4,7}$  identify  $\gamma_6, \gamma_7$

How would this example look with years instead of ages on the x-axis? The real data start at some year (0 for the sake of exposition) and let's assume that all the firms in the sample are available in the first year 0. The result of this transformation is depicted in figure 10b.

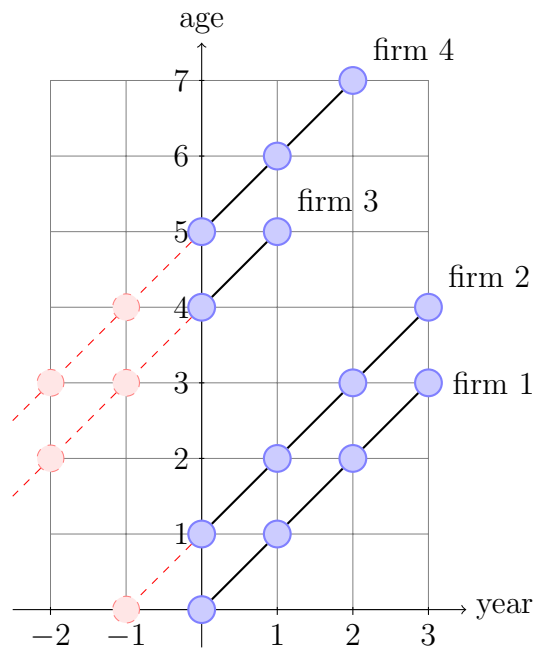
In this example, firm 1 entered in the year 0 and was present until year 3. Firm 2 was already 1 year old at the start of the sample (so it must have entered in year -1, as

Figure 10: Example 3: successful identification

(a) age  $\times$  firm space



(b) year  $\times$  age space (red circles denote the observations that are outside of the sample)



captured by unfilled circle at coordinates  $(-1, 0)$  in the  $(\text{year}, \text{age})$  space) and was present until year 3, in which it reached age 4. The logic is the same for firms 3 and 4.

This example shows that if there are only  $T$  years of data, it is possible to estimate the effect of age  $A$  even if  $A > T$ . This example can be extended to include sector and year effects.